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Inertial core-mantle coupling and libration of Mercury

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ABSTRACT

The internal structure of Mercury is the most puzzling among the terrestrial planets. The space missions MESSENGER and the upcoming BepiColombo as well as ground-based radar measurements will play an important role in constraining our understanding of the structure, formation, and evolution of Mercury. The development of a complete theory of the coupled spin-orbit motion of Mercury within the Solar System is an essential complement to observational data and will improve significantly our knowledge of the planet. Prior work concerning the effect of core-mantle couplings on the rotation of Mercury has assumed that the obliquity of Mercury is equal to zero and that its orbit is Keplerian. This work deals with the Hermean core-mantle interactions in a realistic model of the orbital and rotational motions of Mercury. To this aim, we have used the SONYR model of the Solar System including Mercury's spin-orbit motion (SONYR is the acronym of Spin-Orbit *N*-bodY Relativistic model). We studied the dynamical behavior of the rotational motion of Mercury considered as a solid body including either a solid core or a liquid core. The liquid core and the mantle are assumed to be coupled through an inertial torque on the ellipsoidal core-mantle boundary. We determined Mercury's rotation for a large set of interior structure models of Mercury to be able to identify and to clarify the impact of the core motion on the librations. In this paper, we present a comparative study of the librations resulting from different models of the internal structure. The geophysical models have been calculated for a three-layer planet composed of a solid mantle, a liquid outer core, and a solid inner core. We find that (i) the influence of inertial coupling is of the order of a milliarcsecond for a core ellipticity of the order of 10^{-4}; (ii) the amplitude of the 88-day libration depends essentially on the radius of the core or, equivalently, on the concentration of sulfur in the core; and (iii) the range of amplitude values is 19 arcsec, indicating the possibility to discriminate between models of internal structure by using accurate libration measurements.

Key words. methods: numerical – celestial mechanics – planets and satellites: individual: Mercury

1. Introduction

The internal structure of Mercury is the most puzzling among the terrestrial planets, and the state and dimension as well as the composition of its core are open problems. Mercury has the highest uncompressed density implying a very large core consisting mainly of iron. A simple two-layer model of the interior of Mercury gives a core radius of about 1900 km, implying a radius ratio $r_{\rm core}/r_{\rm planet} \approx 0.75$. This value is large compared to the values for Venus, the Earth, and Mars, which are about 0.5. The bulk composition of Mercury is thus also anomalous, for example the bulk iron/silicon ratio is estimated to be about twice that of the other terrestrial planets (Balogh & Giamperi 2002). The discovery of the magnetic field of Mercury by Mariner 10 (Ness et al. 1975) suggests that at least an upper layer of the core is still liquid, and generating a magnetic field by dynamo action, although it can not be ruled out that the observed magnetic field is due to other sources, such as a remanent crustal field (Aharonson et al. 2004). Since a pure iron (Fe) core could not have remained molten due to the cooling of the planet since its origin, a small amount of sulfur (S) is often introduced to depress the freezing temperature of the core alloy (Schubert et al. 1988).

The origin and the amount of sulfur in Mercury are not well understood. If Mercury was formed by condensation from the solar nebula at its present position, the planet should be strongly depleted in volatiles (such as sulfur) and the core would contain only a very small concentration of sulfur (Lewis 1988). An alternative explanation for the presence of sulfur is that Mercury was not formed at its present orbital distance but at a distance further away from the Sun (Wetherill 1980). Migration to its present position could have happened after a catastrophic giant impact that removed a substantial part of the mantle (Wetherill 1988). Recent simulations of the formation of terrestrial planets suggest that a non-negligible amount of light elements can be accreted from the source region between Mars and Jupiter (O'Brien et al. 2006). The determination of the state, the size, and the chemical composition of the core will provide crucial constraints for the formation as well as for the evolution of Mercury and more generally of terrestrial planets.

Recent observations performed by Margot et al. (2004) seem to prove that the core is liquid. These authors implemented a new radar technique based on cross-correlation of planetary speckle patterns observed at two distant sites on Earth. With this interferometric technique, they obtain two observables, the time delay and the epoch associated with maximum correlation, both of which constrain the rotation of Mercury with high precision. These observational data combined with future data from the space missions devoted to Mercury, namely MESSENGER and BepiColombo, will provide unprecedented constraints on the internal structure by measuring in particular two rotational parameters of Mercury, the obliquity η and the angle of libration in longitude φ, with a great accuracy (Solomon et al. 2001;

Milani et al. 2001). The obliquity is the angle defined by the polar axis and the orbital pole. The so-called libration in longitude of 88 days corresponds to the rotation variations around the figure axis at the 88-day period (the orbital period of Mercury). The amplitude of this libration depends not only on the influence of the solar torque on Mercury, but also on the internal structure of the planet.

Following the discovery of the 3:2 spin-orbit resonance (Pettengill & Dyce 1965; Colombo 1965), the rotation of Mercury has been studied during the second part of the 1960's in the framework of the Cassini theory (e.g. Colombo 1966; Peale 1969, 1972, 2005; Beletskii 1972). The spin-orbit motion of Mercury is characterized by (i) a 3:2 spin-orbit resonance between cyclic variables (rotation and mean motion) and (ii) a synchronism between precession variables (precessional motion of the equator plane and the orbital plane). The upcoming space missions to Mercury stimulated the developement of new theories. Rambaux & Bois (2004) have identified and analyzed the main librations of Mercury as well as the dynamical behavior of the obliquity. They used a relativistic gravitational model of the Solar System including the spin-orbit motion of Mercury; this model is called SONYR, acronym of Spin-Orbit *N*-bodY Relativistic model (details in Sect. 2). In Rambaux & Bois (2004), Mercury is considered as a rigid solid body. A Hamiltonian approach of the rotational motion of Mercury has been expanded by D'Hoedt & Lemaître (2004). The orbit is taken Keplerian and the planet is also a rigid solid body. The values of the two frequencies related to the spin-orbit resonance of Mercury determined with the SONYR model and by the analytical approach have been found to be in good agreement. The libration at these frequencies is often called free libration (e.g. Peale 2005). However, the terminology of rotational dynamics is not uniform, especially in the case of non-rigid bodies, and these frequencies are also referred to as proper frequencies in the literature (see details in Bois 1995).

The first method for obtaining constraints on the state and structure of the Hermean nucleus is based on the assumptions introduced by Peale in 1972 and revisited in 2002 (Peale et al. 2002). In this framework, the rotation is described for a zero obliquity as well as a Keplerian orbit; it is assumed moreover that the solid upper layer (the mantle) is decoupled from the interior by a liquid layer at the libration timescale of 88 days. In this case, the libration in longitude is found inversely proportional to the largest moment of inertia of the solid upper layer, which can therefore be deduced from libration measurements (Peale 1972). Peale et al. (2002) have also estimated several core-mantle coupling mechanisms and in accordance with their assumptions concluded that these mechanisms are most likely too small to noticeably influence the mantle libration.

In this paper, we take into account the existence of another core-mantle coupling. We consider the inertial coupling between the liquid core and the solid mantle of Mercury related to the triaxial ellipsoidal shape of the core-mantle boundary. The dynamical motion of a core inside the elliptical mantle is numerically integrated in the full spin-orbit motion of Mercury included in the whole Solar System. We determine Mercury's rotation for a large set of interior structure models of Mercury to be able to identify and to clarify the impact of the core motion on the librations.

The paper is organized as follows. Section 2 describes our spin-orbit integration method and the inertial coupling between the core and mantle. The internal structure models used are briefly discussed in Sect. 3. Section 4 is devoted to the libration results and deals with the signature on the 88-day libration of various internal structure parameters. Conclusions are presented in Sect. 5.

2. The SONYR model

2.1. The framework of the model

An accurate theory of Mercury's spin-orbit motion included in the Solar System has been constructed. It is based on the BJV model of the Solar System dynamics including the coupled spin-orbit motion of the Moon (Bois 2000; Bois & Vokrouhlický 1995). This model, expanded in a relativistic framework, has been built in accordance with the requirements of the Lunar Laser Ranging observational accuracy. Rambaux & Bois (2004) extended the BJV model by including the spin-orbit couplings for the four terrestrial planets, and called the model *SONYR* (acronym of Spin-Orbit *N*-Body Relativistic model). The SONYR model is used here as it performs an accurate simultaneous integration of the spin-orbit motion of Mercury. The approach of the BJV and SONYR models consists in integrating the *N*-body problem (including translational and rotational motions) based on general relativity. The equations have been developed in the DSX formalism presented in a series of papers by (Damour et al. 1991, 1992, 1993). For purposes of celestial mechanics, to our knowledge, it is the most suitable formulation of the post-Newtonian (PN) theory of motion for a system of *N* arbitrarily extended, weakly self-gravitating, rotating and deformable bodies in mutual interactions. The DSX formalism, derived from the first PN approximation level, includes both the translational and rotational motions of the bodies with respect to frames locally transported with the bodies. Gravitational figures of the bodies are represented by complete expansions in spherical harmonics (Borderies 1978; Schutz 1981). The SONYR model is described in detail in Rambaux & Bois (2004).

The SONYR code is modular and specific physical effects can be studied individually. Each module, containing the physics, can be activated or not according to the problem at hand. With the modular approach, the model can also readily be extended with hitherto neglected dynamical or geophysical effects. The model, solved by numerical integration, is ideally suited for a systematic analysis of all the effects and contributions, and for the identification of relationships between causes and effects. In particular, the various families of lunar and Hermean librations can be identified and isolated as shown in Bois & Girard (1999); Bois (2000); Bois & Vokrouhlický (1995); as well as in Rambaux & Bois (2004).

We introduce a global reference frame $O'X'Y'Z'$ centered on the Solar System barycenter, the $X'Y'$ plane is parallel to the J2000 ecliptic plane. The X'-axis is oriented to the equinox *J*2000. The rotational motion of Mercury is evaluated from a body-fixed coordinate axis system $Oxyz$ centered on Mercury's center of mass relative to a local dynamically non-rotating reference frame, $OXYZ$. The Ox, Oy, Oz axes are defined as the principal axes of inertia. In the framework of the present paper without purely relativistic contributions, the $OXYZ$ local reference frame, although falling down in the gravitational field of the Sun, is not affected by a slow (de Sitter) precession of its axes transported with the translational motion of Mercury. The axes of $OXYZ$ are chosen parallel to those of $OX'Y'Z'$. We used the Euler angles ψ, θ, φ related to the 3-1-3 angular sequence (3 represents a rotation around a Z-axis and 1, around an X-axis) to describe the evolution of the body-fixed axes $Oxyz$ with respect to the axes of the local reference frame $OXYZ$. These angles are described in Fig. 1 and are defined as follows: ψ is the

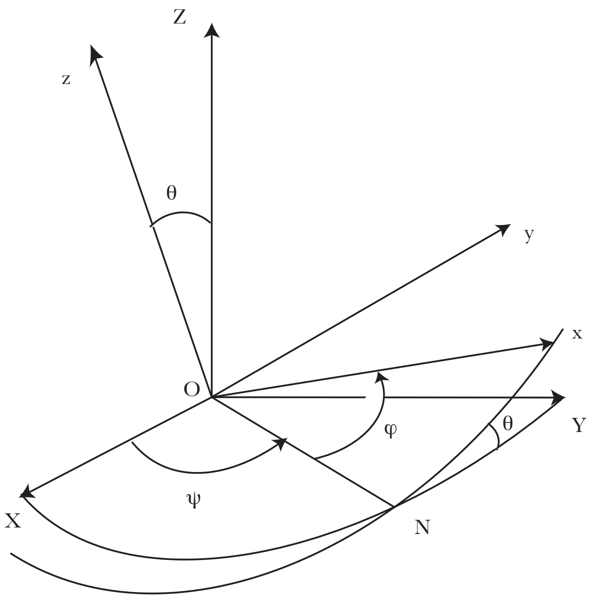

Fig. 1. Description of the Euler angles ψ, θ, φ. $OXYZ$ is the local reference frame and $Oxyz$ is the body-fixed reference frame. O is the center of mass of Mercury.

precession angle of the polar axis Oz around the reference axis OZ, θ is the nutation angle representing the inclination of Oz with respect to OZ, and φ is the rotation around Oz and conventionally understood as the rotation of the largest rotational energy. This angle is generally called the proper rotation. In the literature the angle φ, without its secular part of 58.646 days related to the uniform rotation, is called the libration angle in longitude. The principal axis Oz is called the axis of figure and defines the North pole of the rotation (Bois 1995). In this paper, we concentrate on the dynamical behavior of the φ angle of 88-day libration in longitude.

The numerical integration of the Euler-Liouville equations requires coherent initial conditions for orbital and rotational motions. A traditional way for obtaining such consistent initial conditions is to use accurate observations or ephemeris. Our orbital initial conditions are taken from the JPL ephemeris, but rotational initial conditions are not available due to a lack of accurate observations. Here, we follow the method developed by Bois & Rambaux (2007) to find the libration center of the spin-orbit system. With this method one avoids arbitrary amplitudes in the librations. In a first step of the method, mean initial conditions are determined that verify geometrical conditions of a Cassini state for Mercury, which corresponds to a spin-orbit equilibrium state. In a second step, the mean initial conditions are fitted in order to locate the spin-orbit system at its center of libration. For the φ angle, the first step consists in locating the planet's longest axis of inertia such that it points toward the Sun at each perihelion passage of Mercury (Colombo & Shapiro 1966). In the second step, we fit the mean initial rotation rate $\langle \mathrm{d}\varphi/\mathrm{d}t \rangle$ such that:

$$\langle \mathrm{d}\varphi/\mathrm{d}t \rangle = 3/2n + \langle \omega \rangle \tag{1}$$

where ω is the argument of the pericenter (which presents variations in the N-body problem), and the symbol $\langle . \rangle$ denotes the mean value over 88 days. With this double step procedure, we succeed in avoiding arbitrary amplitudes in the libration angles (a complete demonstration and proof can be found in Bois & Rambaux 2007).

The present paper is devoted to the study of the signature of core-mantle coupling on rotation, the translational and rotational motions being simultaneously integrated. The translational motion is described by a N-body problem (Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and Neptune) in which the planets are assumed point-like masses except for Mercury, which its gravity field is expanded in spherical harmonics up to the degree 2. Moreover, in this study, we focus on librations acting at the 88-day frequency and related to the presence of a core.

2.2. Extension: inertial coupling

If the liquid core can be assumed not to follow the mantle libration, and the obliquity is considered to be zero, the amplitude of libration in longitude of 88 days is given by:

$$A_\varphi = \frac{3}{2}\frac{B-A}{C_\mathrm{m}}(1 - 11e^2 + 959/48e^4 + ...) \tag{2}$$

where e is the eccentricity and C_m the moment of inertia of the mantle (Peale 1972). A and B are the equatorial principal moments of inertia of Mercury ($A < B$). Peale et al. (2002) estimated the possible effects of various coupling mechanisms between mantle and core on Mercury's 88-day libration. In particular, these authors studied the coupling due to pressure forces on the core-mantle boundary (CMB) topography, and showed it to be very small. It is thus reasonable to ignore topography with small angular scales and to concentrate on the pressure effects on the triaxial ellipsoidal CMB. The classical problem of motion of a liquid in an ellipsoidal cavity has a long history. A quasi-rigid rotational motion of the liquid core has been proposed by many authors (e.g. Sasao et al. 1981; Touma & Wisdom 1993), and is often referred to as Poincaré motion (1910), who introduced this notion of "simple motion" in a particularly simple and enlightening way.

The Poincaré motion $\boldsymbol{v}$ of the liquid core can be expressed as:

$$\boldsymbol{v} = \boldsymbol{\omega}^f \times \boldsymbol{r} + \boldsymbol{v}^r \tag{3}$$

where $\boldsymbol{\omega}^f$, the rotation velocity of the core, is independent of position, and $\boldsymbol{v}^r$ is a residual motion, which is smaller than the rigid rotation $\boldsymbol{\omega}^f \times \boldsymbol{r}$ by a factor proportional to the flattenings (see definitions of polar and equatorial flattenings Eqs. (5) and (6)) of the core-mantle boundary. The fluid exerts a pressure on the CMB, and the resulting pressure torque is proportional to the flattenings. The total interaction torque at the CMB includes the gravitational effects and can be expressed in terms of the differential rotation $\boldsymbol{\chi}$ between the core and the mantle. In this case, the rotation of a solid planet with a liquid core can be described by two sets of equations expressing the changes in angular momentum of the whole planet and the core, respectively (Moritz & Mueller 1987):

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}(A\omega_1 + F\chi_1) - \omega_3(B\omega_2 + G\chi_2) + \omega_2(C\omega_3 + H\chi_3) = M_1\\
&\frac{\mathrm{d}}{\mathrm{d}t}(B\omega_2 + G\chi_2) - \omega_1(C\omega_3 + H\chi_3) + \omega_3(A\omega_1 + F\chi_1) = M_2\\
&\frac{\mathrm{d}}{\mathrm{d}t}(C\omega_3 + H\chi_3) - \omega_2(A\omega_1 + F\chi_1) + \omega_1(B\omega_2 + G\chi_2) = M_3
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}(F\omega_1 + A_\mathrm{c}\chi_1) + \chi_3(G\omega_2 + B_\mathrm{c}\chi_2) - \chi_2(H\omega_3 + C_\mathrm{c}\chi_3) = 0\\
&\frac{\mathrm{d}}{\mathrm{d}t}(G\omega_2 + B_\mathrm{c}\chi_2) + \chi_1(H\omega_3 + C_\mathrm{c}\chi_3) - \chi_3(F\omega_1 + A_\mathrm{c}\chi_1) = 0\\
&\frac{\mathrm{d}}{\mathrm{d}t}(H\omega_3 + C_\mathrm{c}\chi_3) + \chi_2(F\omega_1 + A_\mathrm{c}\chi_1) - \chi_1(G\omega_2 + B_\mathrm{c}\chi_2) = 0
\end{aligned}$$

where M_i are the components, with respect to the body-fixed axes $Oxyz$, of the external torque acting on the planet, ω_i and χ_i represent the three components of $\boldsymbol{\omega}$ and $\boldsymbol{\chi}$, the rotation velocity of the planet and the differential rotation of the core with respect to the mantle, respectively ($\boldsymbol{\omega}^f = \boldsymbol{\omega} + \boldsymbol{\chi}$). The coefficients ($A$, B, C) and (A_c, B_c, C_c) are the principal moments of inertia for the whole planet and for the core, respectively. The coefficients (F, G, H) are coupling terms that depend on the core mass M_c and on its three axes of lengths a, b, and c, as follows:

$$F = \frac{2}{5}M_c bc, \quad G = \frac{2}{5}M_c ca, \quad H = \frac{2}{5}M_c ab.$$

The larger the CMB flattenings are, the larger the effect of inertial coupling on Mercury's libration will be. The polar and equatorial flattenings of Mercury are very small ($J_2 = 6.0\pm2.0\times10^{-5}$, $C_{22} = 1.0 \pm 0.5 \times 10^{-5}$, Anderson et al. 1987) and are used here to define lower limits for the CMB flattenings (see Sect. 3.2 for details). Much larger CMB flattenings can result from mantle convection, which is generally considered to be the dominant mantle heat transfer mechanism for Mercury (Stevenson et al. 1983; Spohn 1991), although some studies show that convection has ceased in Mercury's mantle (e.g. Reese et al. 2002). Typical horizontal wavelengths of the convection cells in mantles of terrestrial planets are somewhat larger than the depth of the convecting mantle (Schubert et al. 2001), and the range of wavelengths increases with increasing Rayleigh number. It is unsure that the Rayleigh number is larger than the critical Rayleigh number (the necessary condition for convection) for the lowest degree spherical harmonics in the rather thin Mercurian mantle. It depends, among others, on the amount of radioactive elements still present in the mantle. Redmond & King (2005) found some evidence of long-wavelength convection in Mercury's mantle. In another numerical study of mantle convection in Mercury, Conzelmann & Spohn (1999) obtained wide convection cells with horizontal wavelengths much larger than the depth of the mantle of Mercury for a mantle rheology like the Earth's mantle.

Mantle convection can cause large undulations in the CMB of the order of a few kilometers (e.g. Defraigne et al. 1996). For Mars, due to the equatorial location of the Tharsis province, mantle convection can increase the difference in radius between equatorial and polar radius of the core-mantle boundary by up to 5 km (Defraigne et al. 2001). For the Earth, the polar flattening of the core-mantle boundary has accurately been obtained from nutation studies and is found to correspond to an equatorial axis of about 500 m longer than expected from hydrostatic equilibrium (Gwinn et al. 1986; Mathews et al. 2002). For Mercury, large scale organization of the mantle convection can not be excluded a priori, and the CMB flattenings could be largely amplified with respect to the rotational polar flattening, or the surface flattening estimates. As an upper bound to the core-mantle boundary flattenings, we assume that two principal axes can differ by up to a few km.

3. Internal structure models

3.1. MIS models

A series of models of internal structure of Mercury, hereafter called MIS *a*, MIS *b*...MIS *n* (MIS for "Model of Internal Structure"), have been developed in order to study the effect of the core of Mercury on its librations. We consider Mercury to be composed of three layers, namely a solid mantle, a liquid core and inside a solid inner core. Mercury being a rather small planet, models with layers of uniform density are a very good first-order approximation. All models have a mantle with a mass density of 3500 kg m^{-3}, and the core is assumed to be composed of Fe and S. The initial concentration of sulfur is unknown, and we use models with a range of mean sulfur concentration in the total core between 0.1 and 14 wt% (weigth percentage; see Van Hoolst & Jacobs 2003, for more details on the models). All our models satisfy Mercury's mass and radius. A model is uniquely defined by the core sulfur concentration and the inner core radius R_{ic}. For each model we calculated the mean moment of inertia of the core, namely I_c, and for the whole planet, namely I. Let us recall that $I = (A + B + C)/3$ where (A, B, C) are the three principal moments of inertia of the planet.

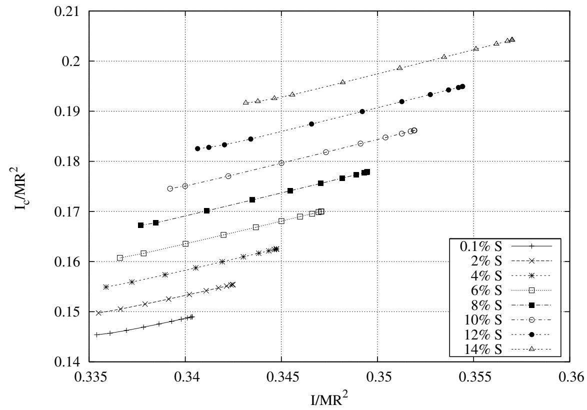

Fig. 2. Mean moment of inertia of the core as a function of the mean moment inertia for the whole planet from the MIS model. The curves are characterized by different sulfur concentrations.

In our dynamical SONYR calculations, the rotation of the mantle is coupled to that of the liquid core by inertial coupling only. Although our MIS models can include an inner core, we consider as a first step that the rotational behavior of the core does not differ from that of an entirely liquid core with the same moment of inertia I_c equal to the sum of the inner and outer core moments of inertia. The inner core is therefore assumed to be locked to the rotation of the outer core, an hypothesis that is often used for the Earth. In reality, the inner and outer cores might not be aligned and the inner core could also be gravitationally coupled to the mantle. Note that Peale et al. (2002) have estimated the effect of this coupling on libration to be negligible.

Figure 2 shows the ranges of the normalized mean moments of inertia I_c/MR^2 and I/MR^2. As Mercury has a very large core, the core's moment of inertia can be seen to be about half of the total moment of inertia.

3.2. Principal moments of inertia

The interior structure models are spherically symmetric, but for our libration calculations, we assume the core and the planet to be of triaxial ellipsoidal form. The three normalized principal moments of inertia $(A/MR^2, B/MR^2, C/MR^2)$ of our models are calculated from the observed J_2 and C_{22} values and from the mean moment of inertia I derived from the models:

$$\frac{C}{MR^2} = \frac{I}{MR^2} + \frac{2}{3}J_2$$
$$\frac{A}{MR^2} = -J_2 - 2C_{22} + \frac{C}{MR^2}$$
$$\frac{B}{MR^2} = -J_2 + 2C_{22} + \frac{C}{MR^2}.$$

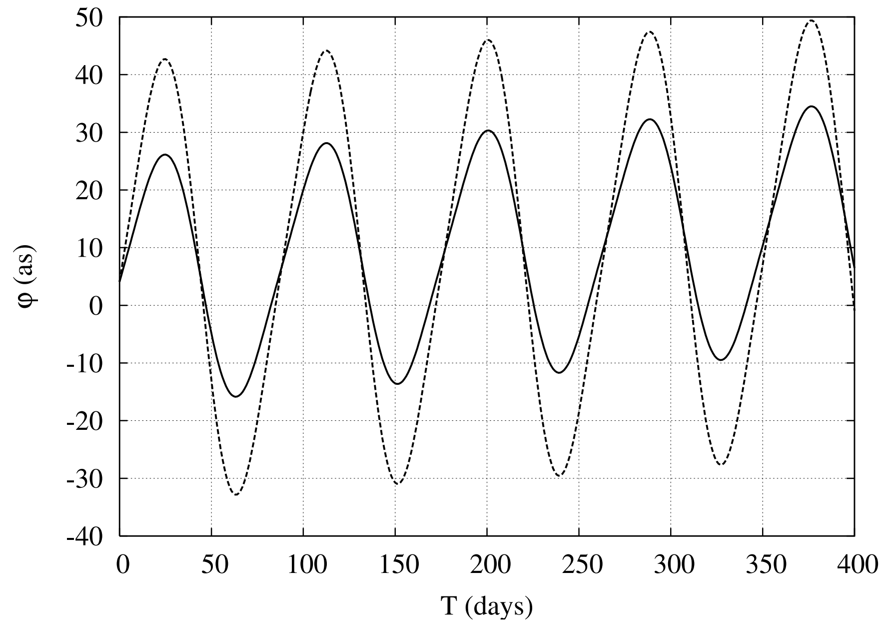

Fig. 3. Libration in longitude for two different internal structures of Mercury. The dashed line represents the case of a body with two layers, the solid line is for a solid body. The curves are plotted without their respective mean proper rotation over 400 days (the expected time for BepiColombo mission). Arcseconds are on the vertical axis.

For the core, there are no observational constraints on the moments of inertia nor on its oblateness. By definition, the polar and equatorial core flattenings, α_c and β_c respectively, are expressed by the following relations:

$$\alpha_c = \frac{C_c - \frac{A_c+B_c}{2}}{\frac{A_c+B_c}{2}}, \quad (5)$$

and

$$\beta_c = \frac{B_c - A_c}{A_c}\cdot \quad (6)$$

We assume the dynamical figure of Mercury's core to be homothetic to the total dynamical figure. Therefore, we put:

$$\beta_c = \beta \quad (7)$$

and

$$\alpha_c = \alpha \quad (8)$$

where α and β are the polar and equatorial flattenings of the whole planet, which are of the order of 10^{-4}. For any model, the three principal moments of inertia can then be calculated from the core mean moment of inertia I_c, and the polar and equatorial core flattenings from:

$$\begin{aligned} A_c &= 3I_c\left[1+\frac{(2+\beta)}{2}(2+\alpha)\right]^{-1}, \\ B_c &= A_c(1+\beta), \\ C_c &= \frac{A_c}{2}(2+\beta)(1+\alpha). \end{aligned} \quad (9)$$

The polar flattening has also been calculated from Clairaut's equation, which gives the flattening due to rotation of a planet in hydrostatic equilibrium. The hydrostatic flattening is, however, two orders of magnitude smaller than that deduced from the observed J_2 and C_{22} values and I model value. Given this indication of mantle and crust non-hydrostaticity and the fact that mantle convection effects on the form of the CMB can not a priori be excluded, we also considered models with flattenings equal to 10^{-3}, corresponding to differences in the principal core axes of a few kilometers. The coupling terms F, G and H, introduced in Sect. 2.2, are calculated from the following expressions:

$$\begin{aligned} F^2 &= A_c^2 - (C_c - B_c)^2, \\ G^2 &= B_c^2 - (A_c - C_c)^2, \\ H^2 &= C_c^2 - (B_c - A_c)^2. \end{aligned} \quad (10)$$

4. Signatures of the core-mantle librations

4.1. 88-day libration in longitude

The dynamical motion of Mercury considered as a solid body has been previously studied in detail in Rambaux & Bois (2004). Using SONYR and the MIS models, we calculate here the dynamical behavior of the rotational motion of Mercury considered as a body with two layers, a liquid core and a solid mantle. Figure 3 shows the evolution over 400 days of the angle of libration in longitude for these both cases. The model with a liquid core has a core radius of 1858 km, a sulfur concentration of 0.1 wt% and a normalized moment of inertia of 0.340 (MIS *a*, see Table 1). The solid model has a moment of inertia of 0.340, corresponding to the value used in Rambaux & Bois (2004).

In order to better distinguish the librations, we have removed the mean rotation of 58.646 days in the φ angle plotted in Fig. 3. A variation with a period of 87.969 days is clearly visible. For the MIS *a* model, the peak-to-peak amplitude of the libration in longitude is about 75 arcsec (as), almost twice as large as the 42 as for the solid Mercury model.

We first analyze the differences between the analytical libration theory and the results obtained by the model SONYR. In the analytical assumptions, the obliquity is constant and equal to zero, and there is no inertial coupling. We introduce these assumptions in our model by computing the two body problem with the figure axis of Mercury normal to the orbital plane (as a consequence the obliquity is constant and equal to zero) and by taking the core spherically symmetric (no inertial coupling). We have found that the relative difference between the analytical solution for the 88-day libration amplitude and the numerical solution is below 0.1% and is due to neglected terms of the analytical formula.

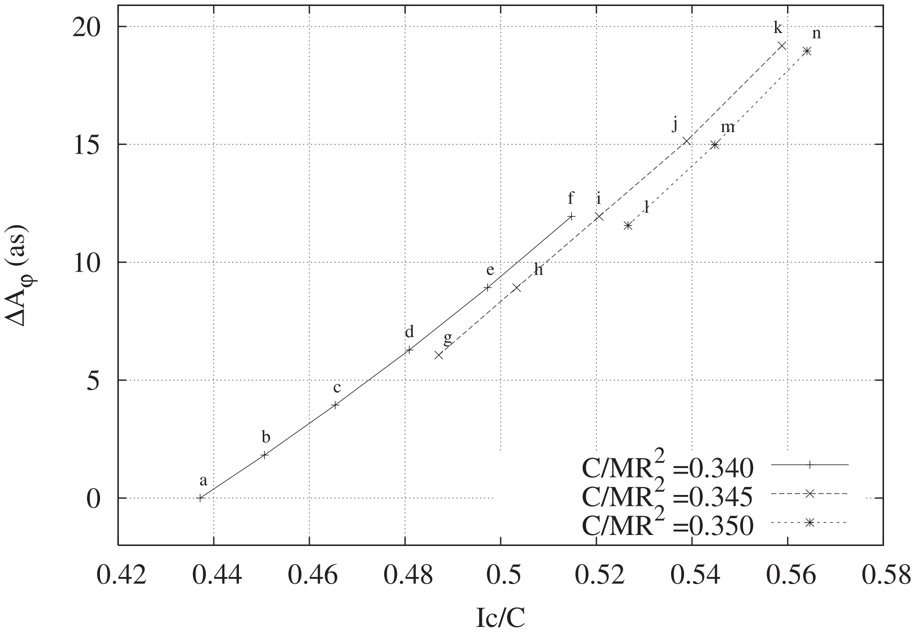

Fig. 4. Signature in the libration amplitude of Mercury for different models of internal structure. The solid line represents the case where $C/MR^2 = 0.34$, the dashed line is for $C/MR^2 = 0.345$, and the dotted line for $C/MR^2 = 0.35$. ΔA_φ (as) is the relative peak-to-peak amplitude obtained as the difference between the numerical integration result for the model considered and the result for the MIS *a* model. The labels on the curves represent the used internal structure models, see Table 1.

Table 1. Interior structure parameters (Van Hoolst & Jacobs 2003). wt% weight concentration of Sulfur in the core, $R_{\rm ic}$ and $R_{\rm c}$ the radii of the inner and outer cores, in kilometers; C/MR^2 and $I_{\rm c}/MR^2$ are the polar principal moments of inertia for the whole planet and for the core, respectively; ΔA_φ (as) is the relative peak-to-peak amplitude obtained as the difference between the numerical integration result for the model considered and the result for the MIS *a* model.

Name	wt% sulfur	$R_{\rm ic}$ (km)	$R_{\rm c}$ (km)	C/MR^2	$I_{\rm c}/MR^2$	ΔA_φ (as)
MIS *a*	0.1	635.000	1858.128	0.3400	0.14863	0.00
MIS *b*	2	1126.000	1877.766	0.3400	0.15321	1.82
MIS *c*	4	1328.000	1899.940	0.3400	0.15824	3.94
MIS *d*	6	1445.000	1923.306	0.3400	0.16350	6.28
MIS *e*	8	1520.000	1947.637	0.3400	0.16906	8.92
MIS *f*	10	1628.000	1971.392	0.3400	0.17504	11.94
MIS *g*	6	907.000	1930.700	0.3450	0.16803	6.06
MIS *h*	8	1130.000	1954.209	0.3450	0.17366	8.72
MIS *i*	10	1263.000	1978.520	0.3450	0.17960	11.70
MIS *j*	12	1355.000	2003.546	0.3450	0.18592	15.14
MIS *k*	14	1557.000	2026.604	0.3450	0.19279	19.18
MIS *l*	10	617.000	1984.002	0.3500	0.18527	11.56
MIS *m*	12	1019.000	2007.843	0.3500	0.19067	14.98
MIS *n*	14	1158.000	2033.123	0.3500	0.19741	18.96

Next, we quantify the effects of core flattenings on the 88-day libration in longitude in a more complete model of the rotation of Mercury (N-body problem). We first consider the MIS a model with a spherically symmetric core and called this case C1. In case C2, the core oblateness $\alpha_{\rm c}$ is equal to 1.703×10^{-4}, i.e. corresponding to its value for the model MIS *a*, and C3 represents an extreme case with $\alpha_{\rm c} = 10^{-3}$. For the three cases, we set $\beta_{\rm c} = 0$.

The comparison of the C1, C2, and C3 cases shows that the impact of core flattenings on the peak-to-peak amplitude of the libration is small, and the relative difference between the C2 and C1 cases is of the order of 0.003%. The model with the largest core oblateness (case C3) presents a relative difference with respect to C1 of 0.020%, i.e. larger than the case C2. The increase in libration amplitude is due to the smaller polar moment of inertia of the mantle $C_{\rm m}$ (see Eq. (2)). This mantle moment $C_{\rm m} = C - C_{\rm c}$ decreases for increasing $\alpha_{\rm c}$ because $C_{\rm c}$ increases, as follows from Eq. (9). The effect of equatorial flattening on libration is of the same order of magnitude as that of the polar flattening.

To be able to study the influence of different internal structure parameters on the libration in longitude, we calculate the libration for a set of different geophysical parameters corresponding to different MIS models. Their model parameter values and libration results are listed in Table 1. We note that the non-linear features of the differential equations make it hard to decorellate effects. Nevertheless, by studying only the impact of a MIS model relative to another one, we obtain the right qualitative behavior of the inertial coupling. The relative peak-to-peak amplitudes ΔA_φ are obtained with respect to the solution of the MIS *a* model. We further note that a fit of the initial conditions on observations only refines slightly the signatures of the 88-day libration given in the paper, which can be considered as upper bounds.

Figure 4 shows the amplitude of the 88-day libration as a function of the ratio $I_{\rm c}/C$ for models with constant values of C/MR^2 successively equal to 0.340, 0.345, and 0.350. The amplitude of libration increases for increasing ratio $I_{\rm c}/C$. To a good approximation, the libration is that of the mantle and its amplitude is inversely proportional to the moment of inertia of the

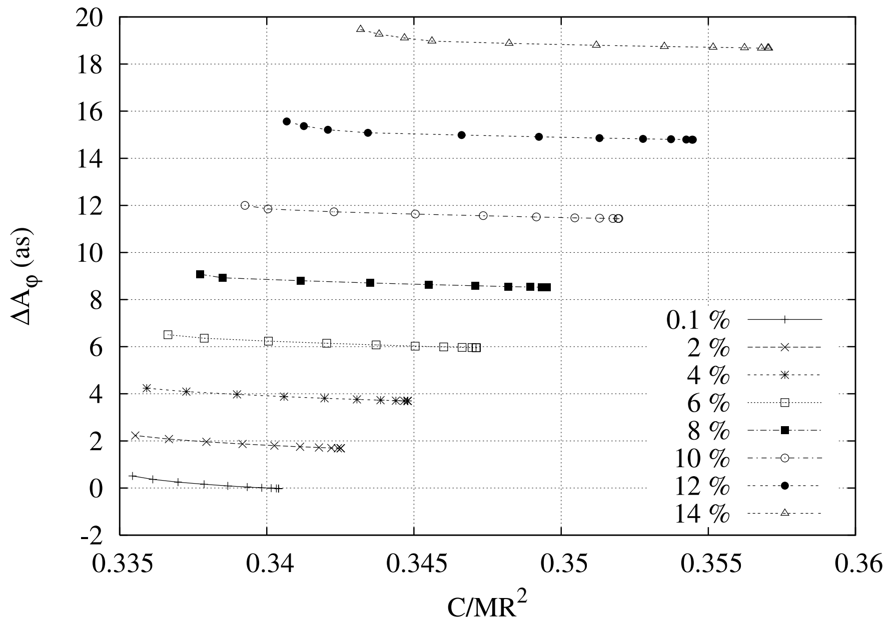

Fig. 5. Signature in the libration amplitude of Mercury as a function of the polar principal moment of inertia for different models of internal structure. ΔA_φ (as) is the relative peak-to-peak amplitude obtained as the difference between the numerical integration result for the model considered and the result for the MIS *a* model. The different symbols are for various sulfur concentrations in the core.

mantle (see Eq. (1)). An increasing core moment of inertia implies a decreasing mantle moment of inertia, and therefore an increasing libration amplitude. The amplitudes range over an interval of 19 as. The range of values is much larger than the expected 3.2 as accuracy forecasted with the measurements of the space mission BepiColombo (Milani et al. 2001), and it will therefore be possible to discriminate between some models of internal structure, to estimate the moment of inertia of the core, and to constrain the chemical composition of Mercury.

The range of values will be even larger for at least two reasons. First, and most importantly, the libration amplitude is proportional to $B - A$, which for all our models is calculated from the Anderson et al. (1987) values for C_{22}. Since this parameter is determined with a 50% error, the libration amplitudes could be about 50% larger or smaller than given here. Nevertheless, the MESSENGER and BepiColombo missions will accurately determine this gravitational coefficients thereby removing this uncertainty in the libration amplitude. Secondly, a more complicated mantle and crust structure than considered here would also change the moments of inertia and therefore the libration amplitude.

Models with three different values for the polar moment of inertia are considered in Fig. 4. The range peak-to-peak relative libration amplitudes for the selected moment of inertia can be expressed as follows:

- $C/MR^2 = 0.340$

  0.00 as $\leq \Delta A_\varphi \leq$ 11.94 as for 1858 km $\leq R_c \leq$ 1971 km
- $C/MR^2 = 0.345$

  6.06 as $\leq \Delta A_\varphi \leq$ 19.18 as for 1931 km $\leq R_c \leq$ 2026 km
- $C/MR^2 = 0.350$

  11.56 as $\leq \Delta A_\varphi \leq$ 18.96 as for 1984 km $\leq R_c \leq$ 2033 km.

These relations are useful for determining the radius of the core of Mercury from the observations of the libration in longitude and the estimation of C/MR^2.

Figure 5 presents the amplitude of libration as a function of the ratio C/MR^2 for all our models. Such a figure highlights the strong dependence of the amplitude of libration on the composition, density of the core, and core radius.

4.2. Long period libration

The combination of ground-based radar measurements and upcoming measurements of the Mercury missions MESSENGER and BepiColombo may allow the detection of the impact of the core on longer time scales up to 20 years (BepiColombo is scheduled for arrived in 2019). The 3:2 spin-orbit resonance of the hermean motion generates a period of 15.85 years for an entirely solid model (Rambaux & Bois 2004). For our models with a liquid core, the long period is between 10.61 and 11.82 years (see Fig. 6). The period is proportional to the square root of the moment of inertia of the mantle alone as the libration is essentially a mantle libration. Therefore, the period decreases for decreasing mantle moment of inertia.

If the long-term libration will be observed by accumulating observations, its period could be used as an additional constraint on the interior of Mercury. We note that dissipation damps the amplitude of the long period libration, and its detection is not ensured (Peale 2005).

4.3. Orientation of the spin axis in space

In analytical libration studies (Peale 1972), the core is assumed not to interact with the obliquity of the planet and the obliquity is assumed to be constant. With the SONYR model, we can follow the motion of the spin axis of Mercury in space and look for the core dynamics. We consider the spin axis position $P = (P1, P2)$ in OXY (J2000 ecliptic frame), defined by:

$$P1 = R \sin\psi \sin\theta$$

$$P2 = -R \cos\psi \sin\theta$$

where R = 2439 km is the equatorial radius of Mercury (Anderson et al. 1987). Figure 7 presents the motion of Mercury's spin axis $(P1, P2)$ plotted over 800 days for the solid core (C/MR^2 = 0.340) and for a liquid core model (MIS *a*). By observing the motion of the spin axis, we can deduce a secular motion of the order of 30 m for MIS *a* and 15 m for the solid case. This shift results from the difference in the precession constant for the two models of internal structure. The difference

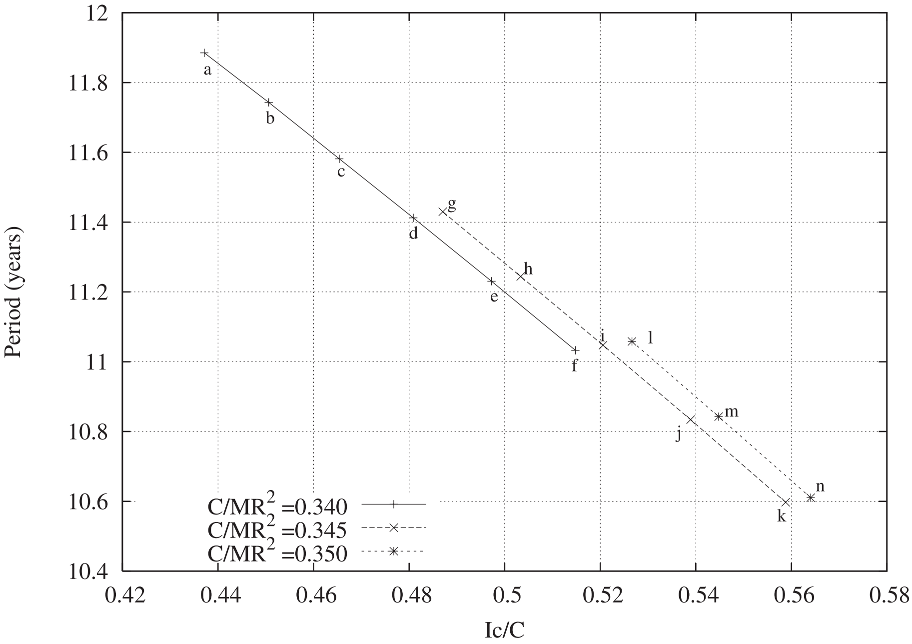

Fig. 6. Period of Mercury's decadal libration as a function of the ratio between the mean moment of inertia of the core I_c and the polar principal moment of inertia C for different models of the internal structure (see the text for details). The labels on the curves represent the internal structure models used, see Table 1.

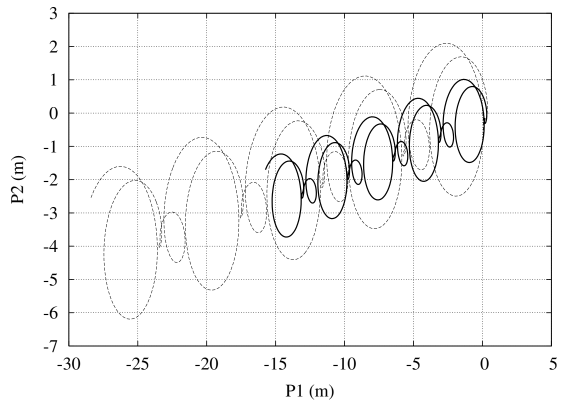

Fig. 7. Motion of the spin axis of Mercury plotted on OXY over 800 days for the homogenous solid case (solid line) and for the two-layer case, MIS *a* (dashed lines).

between these two values is not high enough to distinguishing between the two cases with BepiColombo's measurement accuracy (3.2 as, i.e. 30 m at the surface) for this short-period time, but the accumulation of data during years by different space missions may allow to do so.

We find that the spin axis motion changes periodically with a period of 175.95 days. This motion is due to the solar torque acting on the rotation of Mercury and disturbing the rotation according to the two main frequencies, namely 58.646 and 87.969 days. The difference between the solid and liquid core cases are too small to be observed by the upcoming space missions.

5. Conclusion

We have extended the SONYR model by including the possibility to consider a core and core-mantle couplings inside the terrestrial planets. SONYR then becomes a model at the crossroad of Celestial Mechanics and Geophysics. In particular, SONYR allows to analyze and identify the different families of rotational librations. In this paper, we have studied the *centrifugal librations*, which express signatures of the internal structure of a planet, in particular due to the presence of a core (for details on the terminology of librations, see Bois 1995). We have evaluated the effect of core-mantle inertial couplings and shown the influence of this coupling is of the order of a milliarcsecond for a core ellipticity of the order of 10^{-4}. The changes in obliquity of the planet have been found at the same level. Although these two effects are not negligible, their combination is still below 0.2 as. We therefore extend the conclusions of Peale et al. (2002) by (i) confirming that the core-mantle coupling could be neglected at the first order approximation of the order of a few arcseconds, and (ii) showing for the first time the causal relations between the sulfur concentration and the amplitude of librations for Mercury. Using different interior structure models of Mercury, we have shown that the range of possible libration amplitudes is around 19 arcsec, i.e. is one order of magnitude larger than the observational accuracy of MESSENGER and BepiColombo. Therefore, the observation of the 88-day libration amplitude will allow us to determine not only the state of the core but also to constrain its dimension and composition.

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